

MHD Stagnation Point Flow with Suction Towards a Shrinking Sheet

(Aliran Titik Genangan MHD dengan Sedutan terhadap Kepingan yang Mengecut)

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ABSTRACT

A steady two-dimensional magnetohydrodynamic (MHD) stagnation-point flow of a viscous and electrically conducting fluid over a permeable shrinking sheet has been studied. The governing partial differential equations are reduced to the nonlinear ordinary differential equations by a similarity transformation. The resulting differential equations are then solved numerically using an implicit finite difference method. It is found that the solutions are non-unique for weak magnetic field, strong suction and large velocity ratio between free stream velocity and wall shrinking velocity.

Keyword: MHD; shrinking sheet; stagnation point; suction

ABSTRAK

Aliran titik genangan dua matra mantap magnetohidrodinamik (MHD) terhadap kepingan meregang telah dikaji. Persamaan pembezaan separa menakluk diturunkan kepada persamaan pembezaan biasa tak linear dengan menggunakan penjelmaan keserupaan. Persamaan pembezaan biasa yang terhasil itu kemudiannya diselesaikan secara berangka menggunakan kaedah beza terhingga tersirat. Didapati bahawa penyelesaian adalah tidak unik untuk medan magnet yang lemah, sedutan yang kuat dan nisbah halaju yang besar antara halaju aliran bebas dengan halaju pengecutan dinding kepingan.

Kata kunci: Lapisan mengecut; MHD; sedutan; titik genangan

INTRODUCTION

A class of flow problems with obvious relevance to numerous applications in industrial manufacturing processes is the flow induced by the stretching motion of a flat elastic sheet. Such flow situations are encountered, for example, in aerodynamic extrusion of plastic and rubber sheets, melt-spinning, hot rolling, wire drawing, glass-fiber production, polymer sheets, cooling of a large metallic plate in a bath which may be an electrolyte, etc. During its manufacturing process, a stretched sheet interacts with the ambient fluid both thermally and mechanically. The study of heat transfer and flow field is necessary for determining the quality of the final products of such processes as explained by Karwe and Jaluria (1988, 1991). Crane (1970) was the first who studied the steady two-dimensional incompressible boundary layer flow of a Newtonian fluid caused by the stretching of an elastic flat sheet which moves in its own plane with a velocity varying linearly with the distance from a fixed point due to the application of a uniform stress. This problem is particularly interesting since an exact closed form solution of the two-dimensional Navier-Stokes equations has been obtained. The stability of such flow was shown by Bhattacharyya and Gupta (1985). The uniqueness of the flow has been proved independently by McLeod and Rajagopal (1987), and Troy et al. (1987). After this pioneering work, the flow field over a stretching surface has drawn considerable attention and a good

amount of literature has been generated on this problem (Abraham & Sparrow 2005; Magyari & Keller 2000, Sparrow & Abraham 2005, Wang 1984). A new solution branch for both impermeable and permeable stretching sheets was found by Liao (2007) and Tan et al. (2008), which indicates that multiple solutions for the stretching surfaces are possible under certain conditions.

In recent years, some interest has been given to investigate the flow over a shrinking sheet, where the sheet is stretched toward a slot and it would cause a velocity away from the sheet. A pioneering paper on this problem has been published by Miklavčič and Wang (2006). From the physical grounds vorticity (rotation or non-potential) flow over the shrinking sheet is not confined within a boundary layer, and the flow is unlikely to exist unless adequate suction on the boundary is imposed (Miklavčič & Wang 2006). Fang et al. (2009) extended the problem to unsteady case while Fang and Zhang (2010) obtained an analytical solution for the thermal boundary layers with suction over shrinking sheet. The shrinking sheet problem has also been extended to micropolar fluid (Ishak et al. 2010) as well as magnetohydrodynamic fluid (Fang & Zhang 2009, Noor & Hashim 2009; Noor et al. 2010; Sajid et al. 2008; Sajid & Hayat 2009). Besides the imposition of suction, an added stagnation flow (which contain the vorticity) towards a shrinking sheet make the solution for such fluids to be existed. This idea is first published by Wang (2008)

who found that solutions do not exist for large shrinking rates and may be non-unique in the two-dimensional case. Recently, Lok et al. (2011) extended Wang’s problem to MHD flow where dual solutions exist for small values of magnetic parameter.

The objective of the present study is to analyze the development of the steady boundary layer flow and heat transfer in two-dimensional stagnation-point flow of an incompressible electrically conducting fluid with suction over a shrinking sheet in the presence of a uniform magnetic field. Only the case when the wall temperature varies with the distance along the sheet is considered. In the previous studies, many authors have considered the effect of suction or stagnation flow independently, but nobody has worked on the combination effect of both suction and stagnation flow towards a shrinking sheet. It has to be mentioned here that the two-dimensional MHD boundary layer flow in the region of the stagnation-point on a stretching flat sheet has been investigated by several authors, such as Ding and Zhang (2009), Ishak et al. (2009), Mahapatra and Gupta (2001) and the references cited therein. Therefore, to the best of our knowledge, the results of this paper are new and they have not been published before.

BASIC EQUATIONS

Consider a steady, two-dimensional flow and heat transfer of an incompressible electrically conducting fluid near the stagnation point on a heated shrinking sheet in the presence of a free stream $u_e(x)$ and uniform ambient temperature T_∞ . The wall shrinking sheet velocity is $u_w(x)$, the mass flux velocity is $v_w(x)$ and the wall temperature is $T_w(x)$, which will be defined later. The x - axis runs along the shrinking surface in the direction of motion and the y - axis is perpendicular to it. A uniform magnetic field of strength B_0 is applied in the positive direction of y - axis. The magnetic Reynolds number is assumed to be small, thus the induced magnetic field is negligible. Under these assumptions, the simplified two-dimensional boundary layer equations governing the flow and heat transfer are (Ding & Zhang 2009; Ishak et al. 2009):

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \tag{1}$$

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = u_e \frac{du_e}{dx} + \nu \frac{\partial^2 u}{\partial y^2} + \frac{\sigma B_0^2}{\rho} (u_e - u), \tag{2}$$

$$u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \alpha \frac{\partial^2 T}{\partial y^2}, \tag{3}$$

while the boundary conditions are:

$$\begin{aligned} v &= v_w(x), \quad u = u_w(x), \quad T = T_w(x) \quad \text{at } y = 0 \\ u &\rightarrow u_e(x), \quad T \rightarrow T_\infty \quad \text{as } y \rightarrow \infty. \end{aligned} \tag{4}$$

We assume that $u_e(x) = ax$, $T_w(x) = T_\infty + bx^n$ and $u_w(x) = -cx$ where a, b, c and n are constants with $a > 0, b >$

0 and $c \geq 0$. Thus, we look for a solution of equations (1)-(3) of the following form:

$$\psi = (a\nu)^{1/2} xf(\eta), \quad \theta(\eta) = (T - T_\infty)/(T_w - T_\infty), \quad \eta = (a/\nu)^{1/2} y, \tag{5}$$

where ψ is the stream function defined in the usual way as $u = \partial\psi/\partial y$ and $v = -\partial\psi/\partial x$. Thus, we have:

$$u = a x f'(\eta), \quad v = -(a\nu)^{1/2} f(\eta), \tag{6}$$

where primes denote differentiation with respect to η . Therefore, $v_w(x)$ is given by:

$$v_w(x) = -(a\nu)^{1/2} s, \tag{7}$$

where $s = f(0)$ is the constant mass flux with $s > 0$ for suction and $s < 0$ for injection. Substituting variables (5)-(7) into equations (2) to (4), we get the following ordinary differential equations:

$$f''' + ff'' - f'^2 + 1 + M(1 - f') = 0, \tag{8}$$

$$\frac{1}{Pr} \theta'' + f\theta' - \eta f'\theta = 0, \tag{9}$$

subject to the boundary conditions

$$\begin{aligned} f(0) &= s, \quad f'(0) = -\varepsilon, \quad \theta(0) = 1 \\ f'(\eta) &\rightarrow 1, \quad \theta(\eta) \rightarrow 0 \quad \text{as } \eta \rightarrow \infty. \end{aligned} \tag{10}$$

Here $\varepsilon = c/a$ is the velocity ratio parameter, $Pr = \nu/\alpha$ is the Prandtl number and $M = \sigma B_0^2 / (\rho a)$ is the magnetic parameter.

The physical quantities of principal interest are the skin friction coefficient C_f and the local Nusselt number Nu_x , which are defined as:

$$C_f = \frac{\tau_w}{\rho u_w^2}, \quad Nu_x = \frac{h_x x}{k}, \tag{11}$$

where $\tau_w = \mu(\partial u/\partial y)_{y=0}$ is the wall shear stress, $h_x = q_w/(T_w - T_\infty)$ is the local heat transfer coefficient, $q_w = -k(\partial T/\partial y)_{y=0}$ is the local heat flux, μ is the dynamic viscosity and k is the thermal conductivity. It is easily shown that the skin friction coefficient C_f and the local Nusselt number Nu_x in (11) are given by:

$$Re_x^{1/2} C_f = f''(0), \quad Re_x^{-1/2} Nu_x = -\theta'(0), \tag{12}$$

where $Re_x = u_w(x)x/\nu$ is the local Reynolds number.

NUMERICAL METHOD

The ordinary differential equations (8)-(9) subject to the boundary conditions (10) have been solved numerically using the Keller-box method for some values of the governing parameters, i.e. magnetic parameter M , suction

parameter s and velocity ratio parameter ε . The Keller-box method involves four steps. First, the ordinary differential equations are reduced to a system of first order ordinary differential equations. Next, the resulting system of equations is expressed in the form of a finite difference using central differences. Then, the equations are linearized using Newton's method before putting them in a matrix-vector form. Finally, the resulting linear system of equations is solved along with their boundary conditions by the block-tridiagonal-elimination method. The details procedure of this method can be found in Cebeci and Bradshaw (1988) and Cebeci (2002).

In this paper, a step size of 0.0005 were used and the convergence criterion was set to 5×10^{-7} , which give accuracy to six decimal places. For some values of M , as s or ε increases, multiple solutions were obtained when considering different values of boundary layer thickness, η_∞ . These values of η_∞ are not fixed, but depend on the values of the governing parameters that considered. For

the first solution, the boundary layer thickness η_∞ , ranges between 3 and 6 while for the second solution, the η_∞ was taken from 8 to 25. It was found that the third solution is possible for quite large values of η_∞ (>50).

RESULTS AND DISCUSSION

Table 1 shows the comparison of the initial values $f''(0)$ with those obtained by Wang (2008) and Kimiaefar et al. (2009) for the case when the magnetic field is absent ($M = 0$) and no effect of suction or injection exist ($s = 0$). It is found that the results are in very good agreement. For the sake of brevity, the following results of this paper are limited to a fixed value of $Pr = 0.7$ (air) and $n = 1$ (linearly increasing wall temperature). We expect that the results are qualitatively similar for other values of Pr and n of the same order.

Figures 1 and 2 show the solution regions for the skin friction coefficient $f''(0)$ and the local Nusselt

TABLE 1. Comparison of initial values $f''(0)$ when $M = s = 0$ and some values of ε

ε	Wang (2008)	Kimiaefar et al. (2009)		Present method
	Integration & shooting method	Homotopy analysis method, 20 th -order	4 th -order Runge-Kutta method	Keller-box method
0.25	1.40224	1.402254441	1.40224078	1.402241
0.5	1.49567	1.495670686	1.495671	1.495670
0.75	1.48930	1.489335189	1.48933	1.489298
1	1.32882	1.32888085	1.328824	1.328817
1.15	0			0
1.2465	1.08223			1.082236
	0.116702			0.116702
	0.55430			0.554295

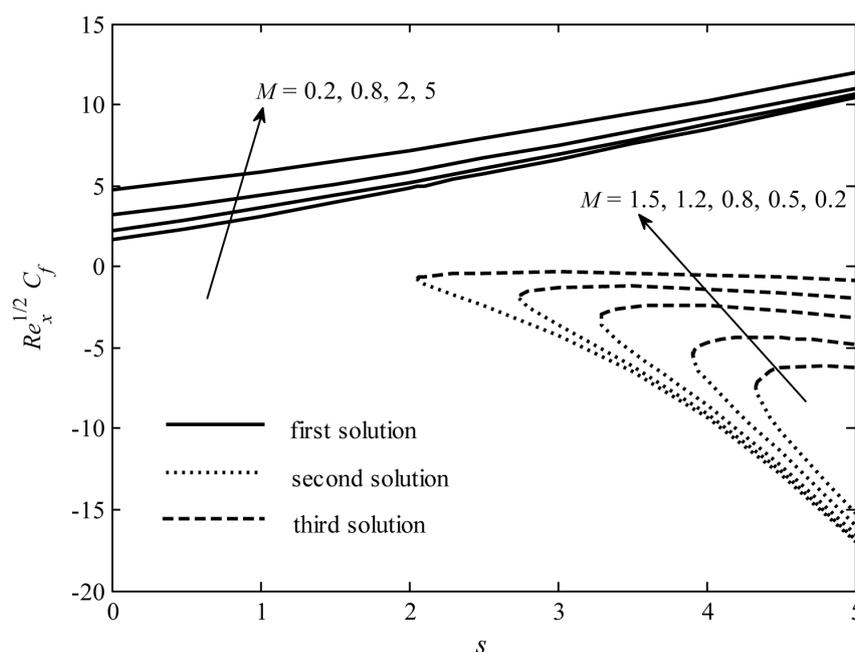


FIGURE 1. Variation of the skin friction coefficient with the suction parameter s for $\varepsilon = 1$ and different values of M

number $-\theta'(0)$ as a function of the suction parameter s for different values of the magnetic parameter M and when $\varepsilon = 1$. It was found that multiple solutions exist beyond the critical value (turning point) s_c for both the skin friction coefficient and the local Nusselt number. Large imposition of suction is required so that multiple solutions are possible for flow with large magnetic parameter. It is found that $s_c = 2.058, 2.745, 3.286, 3.909, 4.332$ for $M = 0.2, 0.5, 0.8, 1.2, 1.5$, respectively. In reality, between these two solutions, only one of them is stable while the other is not. The first solution (solid line) is assumed to be physically stable, because its solution is the continuation of the case of injection ($s < 0$). The second (dot line) and the third solutions (dash line) have negative values of the skin friction coefficient. These solutions show the occurrence of flow separation and reversed flow, which caused the difficulties in the numerical computation. It should be mentioned that Merkin (1985), Weidman et al. (2006), Paullet and Weidman (2007) and Harris et al. (2009) have presented the mathematical proof of the conjecture of dual numerical solutions. They have performed a stability analysis and revealed that the solutions along the upper branch (first solutions) are linearly stable, whilst those on the lower branch (second solutions) are linearly unstable. Besides, from the first (stable) solution in Figure 1, it is observed that $f''(0)$ increases with magnetic parameter M . This is due to the fact that application of a magnetic field to an electrically conducting fluid produces a drag-like force called Lorentz force. Therefore, Lorentz force will be enhanced by increasing M , which imparts additional momentum into the boundary layer (Takhar et al. 2001, Mahmoud 2007). From Figure 2, the first (stable) solution

of the local Nusselt number increases with M , showing that the heat transfer rate increases in the presence of a magnetic field.

Figure 3 represents the velocity $f'(\eta)$, while Figure 4 shows the temperature $\theta(\eta)$ profiles for some values of the suction parameter s and for fixed values of the parameters ($M = 0.5, \varepsilon = 1, n = 1$ and $\text{Pr} = 0.7$). The first and second solutions for large values of s ($s = 3$ and 5) are plotted in these figures as it has been done in Figures 1 and 2 for the skin friction coefficient and the local Nusselt number. The third solution is not shown here as its boundary layer thickness is too large ($\eta_\infty > 50$). From Figure 3, it is found that the velocity profile increases as s increases. This is because suction implies an increase in skin friction coefficient (see Figure 1) which caused by the reduction of momentum boundary layer thickness; hence enhance the flow near the surface of the wall. However, the temperature in Figure 4 decreases as s increases because of the increment of thermal boundary layer thickness. It is also observed that the boundary layer thickness for the second solution is greater than the boundary layer thickness for the first solution. The temperature profiles in Figure 4 for the second solution (dot line) show that negative values are observed near the wall, which represent a physically unrealistic case and should not be happened in reality. This is the reason that we postulate the first solution is physically stable and occur in practice, whilst other solutions are physically not realizable in practice. Further, Figures 5 and 6 investigate the effect of the velocity ratio parameter ε on the velocity $f'(\eta)$ and temperature $\theta(\eta)$ profiles. By taking fixed suction parameter ($s = 3$), it is found that multiple solutions exist when $\varepsilon > 1/2$. It is observed that

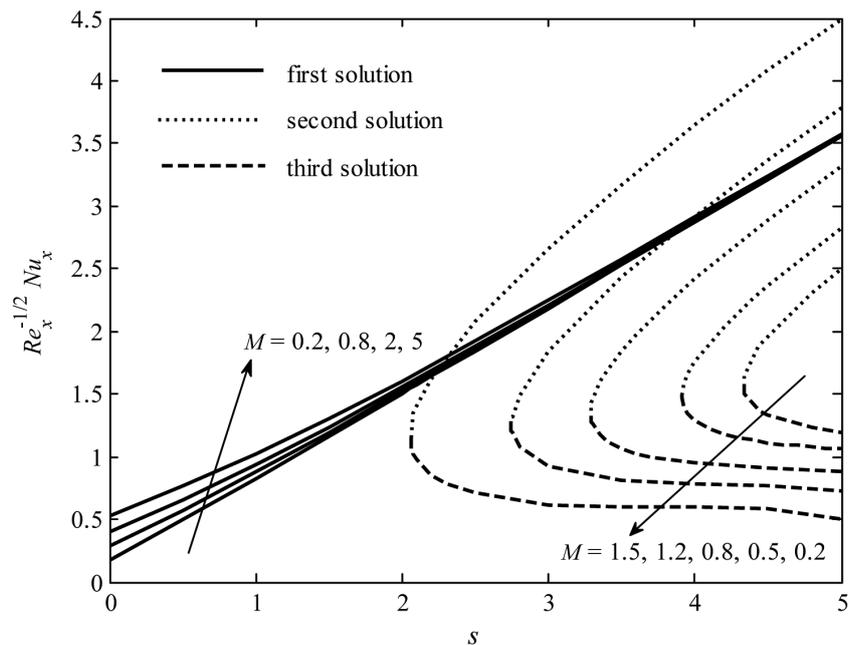


FIGURE 2. Variation of the local Nusselt number with the suction parameter s for $n = 1, \varepsilon = 1, \text{Pr} = 0.7$ and different values of M

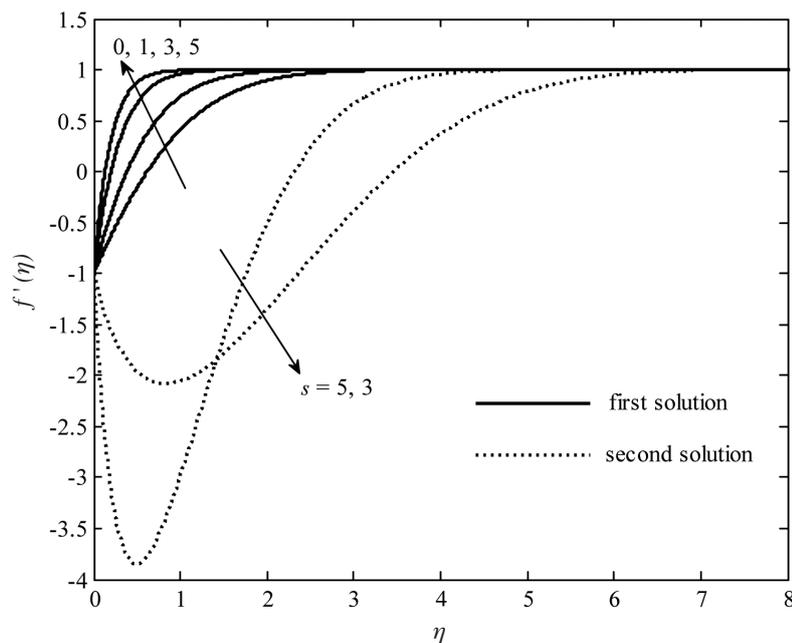


FIGURE 3. Velocity profile for $M = 0.5$, $\epsilon = 1$ and different values of s

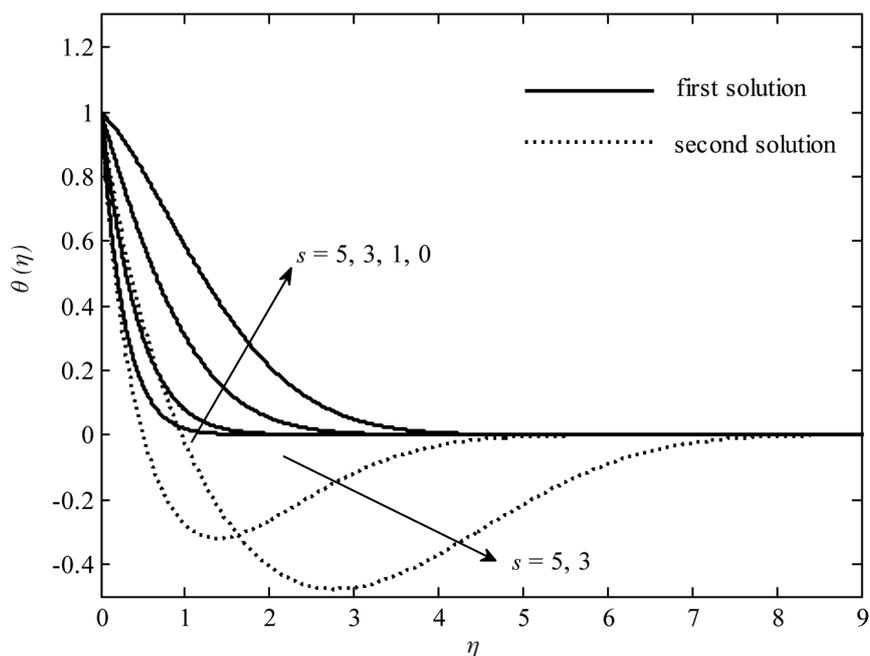


FIGURE 4. Temperature profile for $M = 0.5$, $\epsilon = 1$, $n = 1$, $Pr = 0.7$ and different values of s

for the first solution, the velocity profile increases from its initial value, which is also the minimum value, and then increases to the value of unity. On the hand, for the second solution, the velocity profiles have a negative gradient near the surface of the sheet and then these profiles gradually increase with positive gradient until they reach the far field boundary conditions asymptotically.

It is interesting to see how the streamlines look like for multiple solutions. Thus, the streamlines from (5) for

$M = 0.5$, $\epsilon = 1$ and $s = 3$ are shown in Figure 7. Figure 7(a) represents the streamlines for the first solution, where the pattern is almost similar to the normal stagnation point flow but because of the existence of suction and shrinking effect, the flow is sucked into the permeable wall. The streamlines for the second solution are shown in Figure 7(b). It is found that there exist two horizontal dividing streamlines which separate the flow into three regions. In the upper part, the oncoming flows pass on either side,

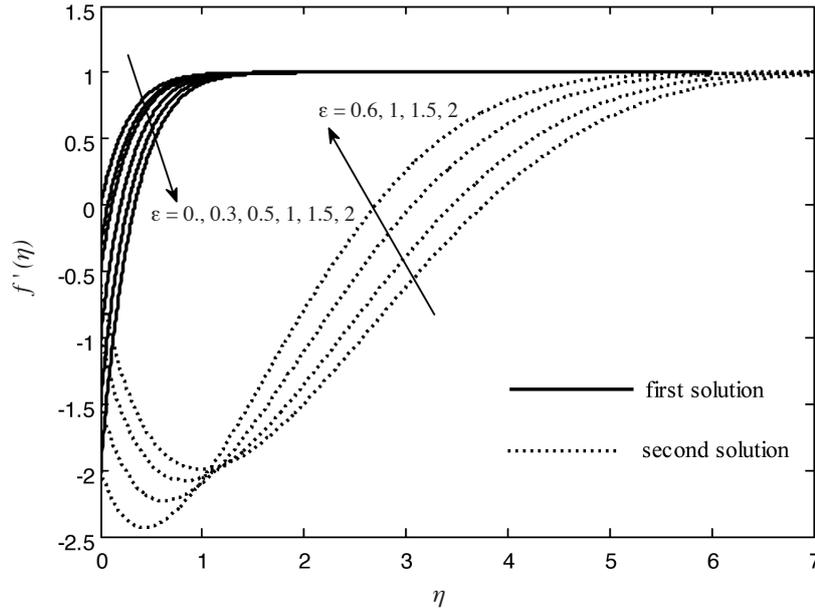


FIGURE 5. Velocity profile for $M = 0.5, s = 1$ and different values of ϵ

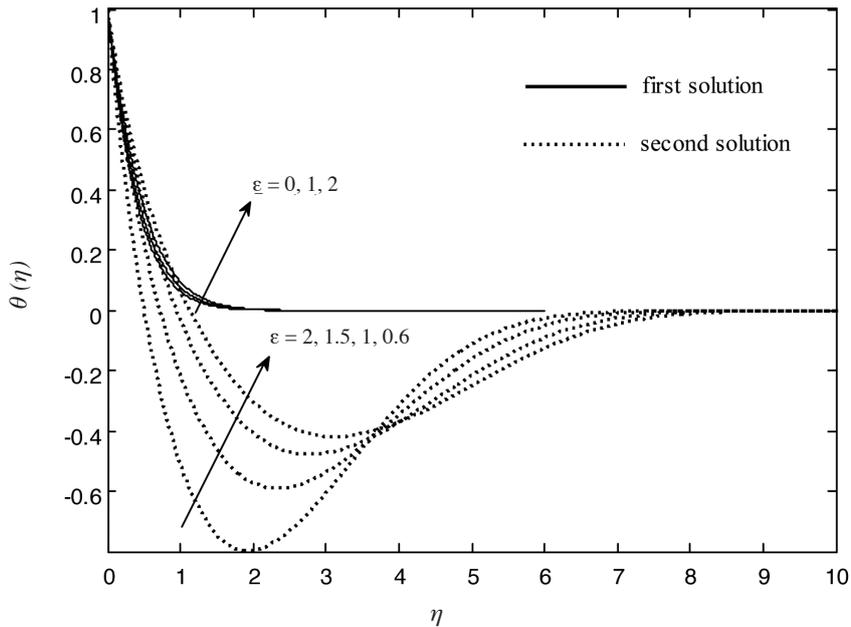


FIGURE 6. Temperature profile for $M = 0.5, s = 3, n = 1, Pr = 0.7$ and different values of ϵ

where the pattern is similar to that of stagnation-point flow. In the second region, reverse rotating (non-potential) flow is formed while in the lowest region, the flows behave like it is dragged to the stagnation point due to shrinking sheet effect. The region of the reverse rotating (non-potential) flow is consistent with the observation of negative velocity gradient in Figure 5. Since we postulate that only the first solution is stable, the streamlines for such case is more simple and controllable.

CONCLUSION

In this paper, the steady MHD stagnation point flow due to a shrinking sheet has been theoretically considered. The effects of suction parameter, magnetic parameter and velocity ratio parameter on the flow and heat transfer characteristics have been studied. The numerical results have been obtained using the Keller-box method. The existence of multiple solutions were observed and

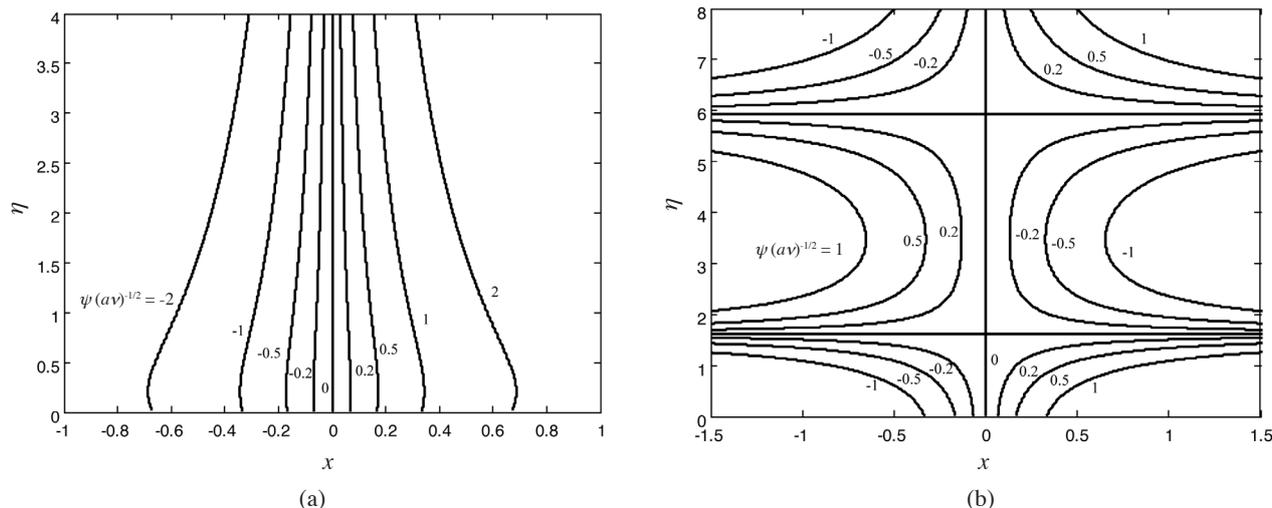


FIGURE 7. Streamlines for the two-dimensional shrinking sheet when $M = 0.5$, $\varepsilon = 1$ and $s = 3$: (a) First solution; (b) Second solution

determined for some values of the governing parameters. It is found that for the first solution (stable solution), both the skin friction coefficient and the local Nusselt number increase as the strength of suction increases. Moreover, strong suction is necessary for the multiple solutions to exist.

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